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Solution of Rayleigh's instability equation for arbitrary wind profiles

By S. BEJI[1] and K. NADAOKA[2]

[1]Department of Naval Architecture and Ocean Engineering, Istanbul Technical University, Maslak 80626, Istanbul, Turkey

[2]Graduate School of Information Science and Engineering, Tokyo Institute of Technology, 2-12-1 O-okayama, Meguro-ku, Tokyo 152-8552, Japan

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A different approach to the solution of the singular Rayleigh equation is presented in the context of the water wave growth problem as modelled by wind-induced shear instabilities. The approach is based on the analytical solution of a Bessel equation in the vicinity of the singular point, which is obtained from Rayleigh's equation with an arbitrary wind profile. Wave growth rates are computed using an integral expression derived from the dispersion relation of the air–sea interface. Computations of the present approach agree well with those of Conte & Miles (1959) for the special case of a logarithmic wind profile. Effects of the shape of the wind profile on the wave growth rate are investigated by using the 1/7-power law to represent the wind profile. Comparisons of the growth rates for the logarithmic wind profile and for the 1/7 profile reveal appreciable differences which must be investigated further, possibly using measured wind profiles within 10 m above the sea surface.

1. Introduction

In his work on the effect of wind on water waves, Kelvin (1871) assumed a frictionless interface, hence a discontinuity of velocities between two fluids, and accordingly determined the conditions of instability. Adopting a similar approach, Rayleigh (1879) studied the instability of jets; however, noting a serious problem in his results concerning the high-frequency motions, Rayleigh (1880) suggested an improvement in the theory by supposing a gradual change in air velocity instead of a discontinuity at the interface. Thus, he derived an equation which is known today as Rayleigh's instability equation or simply the Rayleigh equation. Nevertheless, Rayleigh did not attempt to apply his improvement of the theory to the problem of wave generation by wind.

Quite some time later, Miles (1957) took up the wave growth problem at exactly the point Rayleigh left off. He proposed a model for the growth of wind waves on the basis of Rayleigh's equation with a logarithmic wind profile that was in line with Rayleigh's vision of a gradually changing air velocity. Following Miles (1957), Conte & Miles (1959) presented an accurate method for computing wave growth rates through numerical solution of Rayleigh's equation for the logarithmic wind profile. Since then, the approach of Conte & Miles (1959) has been more or less the only way of tackling this problem numerically.

Miles's model predicts wave growth rates quite commensurate with the observations but somewhat underestimates the growth rate of long waves with phase speeds nearly equal to the wind speed at 10 m height (see Komen *et al.* 1994, p. 72). Numerous attempts to improve the theory by various means have been made. Considering the effects of small-scale turbulence attracted most attention (Gent & Taylor 1976; Al-Zanaidi & Hui 1984; Jakobs 1987; Chalikov & Makin 1991); but results of these works do not differ much from those of Miles. Fabrikant (1976), and later Janssen (1982) advanced the so-called quasi-linear theory of wave generation in which waves modify mean wind velocity in a coupled manner. Further elaborations and applications of this theory were reported by Janssen, Lionello & Zambresky (1989). Nikolayeva & Tsimring (1986) computed substantially enhanced energy transfer due to large-scale turbulence effects, also termed gustiness. On the other hand, starting from the same model, Miles & Ierley (1998) obtained growth rate enhancements considerably less than those predicted by Nikolayeva & Tsimring (1986). A penetrating account of the surface-wave generation by wind, associated problems, and historical details may be found in Miles (1997).

Despite such considerable work on the turbulent modelling and the effects of waves on the drag coefficient, the almost universally accepted logarithmic mean wind profile itself has not been questioned. While quite recent measurements of Powell, Vickery & Reinhold (2003) provide probably the first data confirming a logarithmic wind profile for 10 m to 200 m heights in a marine environment, no measurements exist to confirm the validity of the logarithmic wind profile over sea waves below 10 m height where basically all the wave growth computations show strong influences. Determining the mean wind profile in the range of 0–10 m is obviously a very difficult task, if not impossible. In absence of such measurements it seems plausible to make allowances for the possible variations in the shape of the wind profile rather than strictly adhering to the logarithmic profile.

The present work reconsiders the wind-wave growth problem from a wider perspective and introduces a different approach to solving Rayleigh's instability equation for *arbitrary* mean wind profiles. The solution technique, originally suggested by Rayleigh (1895) himself, is based on re-casting the Rayleigh equation first into a Riccati equation, which is valid strictly in the vicinity of the so-called critical point where the wind speed equals the wave speed. Through appropriate change of dependent and independent variables the Riccati equation is then transformed to a Bessel equation, from which two linearly independent solutions are obtained for the initiation of the numerical integration. Thus, unlike the standard approach of obtaining an approximate series solution of the Rayleigh equation around the critical point, the present approach modifies the Rayleigh equation to obtain a differential equation with well-known analytical solutions.

Wave growth rates are also computed in an unconventional way by implementing the dispersion relation of the air–sea interface, which involves the vertical integration of the disturbed vertical velocity weighted by the wind profile. The integration provides a smoothing effect in the computations and is found to give more reliable results for the 1/7 profile compared to the conventional single-point computation of the wave growth rate. Comparisons of the growth rates with those of Conte & Miles (1959) for the logarithmic wind profile reveal excellent agreement, confirming the accuracy of the method presented.

Taking advantage of the general applicability of the solution technique, the 1/7 power law profile, which is apparently the only alternative wind profile at present, is used for investigating the effects of profile change on the wave growth rate. Plotted growth rates show appreciable qualitative and quantitative differences, pointing out the necessity of studying the actual wind profiles within 10 m above the sea surface.

Further, the effect of coupling between the wind and the surface deformation for different wind profiles should be investigated for realistic conclusions.

2. Rayleigh's instability equation and its solution

2.1. *Rayleigh's equation*

In two dimensions the governing equations of linearized inviscid shear flow for air with a prescribed mean wind velocity $U(z)$ are given by (see Drazin & Reid 1981, p. 127)

$$u_t + U(z)u_x + U'(z)w = -p_x/\rho_a, \tag{2.1}$$

$$w_t + U(z)w_x = -p_z/\rho_a - g, \tag{2.2}$$

$$u_x + w_z = 0, \tag{2.3}$$

where u and w are the horizontal and vertical components of the disturbed velocity, p is the pressure, ρ_a is the density of the air, and g is the gravitational acceleration. A subscript denotes partial differentiation with respect to the indicated variable and a prime stands for differentiation with respect to the vertical coordinate z.

Assuming the horizontal motion periodic in time and in space, the disturbed vertical velocity component is taken as

$$w(x, z, t) = W(z)\exp[\mathrm{i}k(x - ct)], \tag{2.4}$$

where k is the wavenumber, c is the wave celerity, and $W(z)$ is a function of the vertical coordinate z only. From the continuity equation (2.3),

$$u(x, z, t) = (\mathrm{i}/k)W'(z)\exp[\mathrm{i}k(x - ct)]. \tag{2.5}$$

Eliminating pressure by cross-differentiating (2.1) and (2.2) and making use of (2.4) and (2.5) result in Rayleigh's instability equation in terms of $W(z)$:

$$[U(z) - c\,](W'' - k^2W) - U''(z)W = 0. \tag{2.6}$$

The above equation is obviously singular at the critical height $z = z_c$ where $U(z_c) = c$. At present no exact analytical solution of the Rayleigh equation exists; therefore, the usual approach is to resort to a combination of analytical and numerical methods. The first known analytical approximate solution is attributed to Heisenberg (1924) while a well-known form is due to Tollmien (1929) in connection with his work on the Orr–Sommerfeld equation.

2.2. *Solution technique*

Based on Rayleigh's (1895) ideas, a technique is now introduced for obtaining an approximate analytical solution of equation (2.6) around the singular point z_c for an arbitrary wind profile $U(z)$ with non-zero second derivative. First, in the vicinity of the singular point $U(z)$ is approximated by its linearized form while the second derivative of $U(z)$ is replaced by its constant value at z_c:

$$U(z) \simeq U'(z_c)(z - z_c) + c, \qquad U''(z) \simeq U''(z_c). \tag{2.7}$$

These two approximations should be viewed separately and independent of each other since the former does not imply the latter. Using these approximations in equation (2.6) and introducing a change of independent variable as $\tilde{z} = -\,U''(z_c)(z - z_c)/U'(z_c)$ results in the following differential equation which is valid expressly in the close neighbourhood of z_c:

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\tilde{z}^2} + \left[\frac{1}{\tilde{z}} - k^2 \left[\frac{U'(z_c)}{U''(z_c)}\right]^2\right] W = 0. \tag{2.8}$$

Physically, the non-dimensional quantity $\tilde{z} = -U''(z_c)(z - z_c)/U'(z_c)$ may be interpreted as the ratio of the change in the mean vorticity to the mean vorticity and it plays a decisive role in the analytical solution around the critical point.

Near the critical point $\tilde{z}$ approaches zero, hence $1/\tilde{z}$ becomes large in comparison with $[kU'(z_c)/U''(z_c)]^2$; therefore, equation (2.8) may be further approximated as

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\tilde{z}^2} + \frac{1}{\tilde{z}} W = 0, \tag{2.9}$$

which is a Riccati equation. First changing the dependent variable $W = \sqrt{\tilde{z}}\Psi$ and then changing the independent variable again, $\xi = 2\sqrt{\tilde{z}}$, transforms equation (2.9) to

$$\xi^2 \frac{\mathrm{d}^2 \Psi}{\mathrm{d}\xi^2} + \xi \frac{\mathrm{d}\Psi}{\mathrm{d}\xi} + (\xi^2 - 1)\Psi = 0, \tag{2.10}$$

which is a Bessel equation of order one. The two linearly independent solutions of equation (2.10) are given in terms of the Bessel functions of order one:

$$\Psi(\xi) = AJ_1(\xi) + BY_1(\xi), \tag{2.11}$$

where A and B are arbitrary constants. Going back to the original variables gives

$$W(\tilde{z}) = \sqrt{\tilde{z}}[AJ_1(2\sqrt{\tilde{z}}) + BY_1(2\sqrt{\tilde{z}})], \tag{2.12}$$

in which $\tilde{z} = -U''(z_c)(z - z_c)/U'(z_c)$ as defined before. Note that for negative values of $\tilde{z}$ the argument of the Bessel functions is pure imaginary. The above solution, which is valid in the immediate vicinity of the critical point, provides the starting values for the numerical integration of equation (2.6).

The details of the numerical procedure may be described as follows. First, a small quantity, say $\varepsilon \sim 10^{-3} - 10^{-6}$, is selected and just below the critical point $z_{\varepsilon_-} = z_c(1 - \varepsilon)$ is defined. Then, $\sqrt{\tilde{z}}J_1(2\sqrt{\tilde{z}})$ and $[\sqrt{\tilde{z}}J_1(2\sqrt{\tilde{z}})]'$ evaluated at $\tilde{z}_{\varepsilon_-} = -U''(z_c)(z_{\varepsilon_-} - z_c)/U'(z_c)$ supply the starting values for the first linearly independent solution and $\sqrt{\tilde{z}}Y_1(2\sqrt{\tilde{z}})$ and $[\sqrt{\tilde{z}}Y_1(2\sqrt{\tilde{z}})]'$ those for the second solution. In the present problem $\tilde{z}_{\varepsilon_-} < 0$, therefore care must be observed in using the complex-conjugate values of $\sqrt{\tilde{z}}Y_1(2\sqrt{\tilde{z}})$ and $[\sqrt{\tilde{z}}Y_1(2\sqrt{\tilde{z}})]'$ at $\tilde{z}_{\varepsilon_-}$ to ensure a positive growth rate (that is to say, making the appropriate detour about the singularity; see Drazin & Reid 1981, p. 138) as these terms implicity contain the logarithmic singularity. Having prescribed the necessary starting values, equation (2.6) is numerically integrated in the negative direction (below the critical point) for each set of initial conditions between z_{ε_-} and z_0, which is termed the roughness length of the air–water interface and discussed further in §4. The values of linearly independent solutions at the lower limit z_0, as obtained from two separate numerical integrations, are denoted by $W_{J_1}(z_0)$ and $W_{Y_1}(z_0)$, and retained for later use in satisfying the boundary conditions. The second part of the numerical integration proceeds in the positive direction (above the critical point) between $z_{\varepsilon_+} = z_c(1 + \varepsilon)$ and z_∞, which is a relatively large and indefinite upper limit to be determined according to a convergence criterion. The starting values are again provided from the analytical solution as described above but this time evaluated at $\tilde{z}_{\varepsilon_+} = -U''(z_c)(z_{\varepsilon_+} - z_c)/U'(z_c)$. At every integration step in the positive direction the boundary conditions are used to determine the unknown coefficients, say A and B

again, of the desired solution. If the difference in one of the coefficients between two successive steps is less than a specified small value the computation is terminated and the computed values of the linearly independent solutions are denoted by $W_{J_1}(z_\infty)$ and $W_{Y_1}(z_\infty)$. Trial computations show that instead of applying a convergence criterion with an unfixed upper limit of integration it is more convenient and quite sufficient to perform the integration to the fixed height of $\lambda = 2\pi/k$.

The boundary conditions imposed are typical of this kind of problem. Just above the interface at $z = z_0$, a definite value for the vertical velocity, say W_0, is enforced. For great heights, $z = z_\infty$, the disturbances are assumed to vanish: $W' + kW = 0$. Accordingly,

$$A W_{J_1}(z_0) + B W_{Y_1}(z_0) = W_0, \tag{2.13}$$

$$A\left[W'_{J_1}(z_\infty) + kW_{J_1}(z_\infty)\right] + B\left[W'_{Y_1}(z_\infty) + kW_{Y_1}(z_\infty)\right] = 0. \tag{2.14}$$

Solving for the unknown coefficients A and B gives

$$A = \frac{-\left[W'_{Y_1}(z_\infty) + kW_{Y_1}(z_\infty)\right] W_0/W_{Y_1}(z_0)}{\left\{W'_{J_1}(z_\infty) + kW_{J_1}(z_\infty) - \left[W'_{Y_1}(z_\infty) + kW_{Y_1}(z_\infty)\right] W_{J_1}(z_0)/W_{Y_1}(z_0)\right\}}, \tag{2.15}$$

$$B = \frac{\left[W_0 - AW_{J_1}(z_0)\right]}{W_{Y_1}(z_0)}, \tag{2.16}$$

which are either evaluated at every integration step in the positive z-direction till the specified criterion is met at the previously unknown height z_∞, or, if a fixed height is preferred, evaluated only at the pre-selected final height, say $z = \lambda$. Note that since the solution is complex the coefficients A and B are complex too.

3. Dispersion relation of the air–sea interface

Substituting equation (2.4) into (2.2), supposing $p(x, z, t) = P_a(z)\exp[ik(x - ct)]$ for the air pressure and integrating from the sea surface $\eta = a\exp[ik(x - ct)]$ to $+\infty$ gives for $P_a(\eta)$

$$P_a(\eta) = P_0 - \rho_a g a + i\rho_a k \int_{z_0}^{+\infty} [U(z) - c]\, W(z)\, dz, \tag{3.1}$$

where $P_0 = \rho_a g h_\infty$ is the atmospheric pressure at the surface and the lower limit of the integration has been set to z_0 instead of η, since the problem is linearized. $P_a(+\infty)$ is assumed to vanish. For later purposes it is necessary to make use of the kinematic boundary condition at $z = \eta$ for air:

$$\eta_t + U(z)\eta_x = w \quad \text{at} \quad z = \eta. \tag{3.2}$$

Similar to equation (3.1), the above boundary condition is evaluated at the roughness height $z = z_0$ instead of the actual free surface $z = \eta$. Noting that by definition the mean wind velocity $U(z)$ vanishes at $z = z_0$ (see §4) equation (3.2) becomes

$$-ikca = W(z_0) = W_0. \tag{3.3}$$

Using (3.3) in (3.1) gives

$$P_a(\eta) = P_0 - \rho_a g a + \rho_a k c^2 a \frac{(k/c)}{W_0} \int_{z_0}^{\infty} [U(z) - c] W(z)\, dz, \tag{3.4}$$

which is the air pressure on the free surface due to the wind.

For the water wave motion it is fairly straightforward to show that for deep-water waves the pressure on the free surface can be expressed as

$$P_w(\eta) = P_0 - \rho_w g a + \rho_w k c^2 a, \tag{3.5}$$

where ρ_w is the density of the water. The dispersion relation of the combined air–water system can then be obtained from the continuity of pressure across the interface; that is, $P_a(\eta) = P_w(\eta)$:

$$P_0 - \rho_a g a + \rho_a k c^2 a \frac{(k/c)}{W_0} \int_{z_0}^{\infty} [U(z) - c]\, W(z)\, \mathrm{d}z = P_0 - \rho_w g a + \rho_w k c^2 a. \tag{3.6}$$

Eliminating P_0, dividing by $\rho_w a$, and solving for c^2 results in

$$c^2 = \frac{g}{k} \frac{(1-s)}{\left[1 - s\dfrac{(k/c)}{W_0} \displaystyle\int_{z_0}^{+\infty} [U(z) - c] W(z)\, \mathrm{d}z\right]}, \tag{3.7}$$

where $s = \rho_a/\rho_w$. Defining a dimensionless complex integral,

$$I_c = \frac{(k/c)}{W_0} \int_{z_0}^{+\infty} [U(z) - c] W(z)\, \mathrm{d}z, \tag{3.8}$$

and noting that $s \simeq 10^{-3}$ is a small quantity, (3.7) may be approximated as

$$c \simeq c_0[(1 - s/2)/(1 - sI_c/2)] \simeq c_0(1 - s/2 + sI_c/2), \tag{3.9}$$

in which $c_0 = \sqrt{g/k}$ is the deep-water wave celerity. Note that both $W(z)$ and c appearing in the integral I_c are complex; but in evaluating the integral the unknown complex phase speed c may be taken approximately real as its imaginary part is proportional to s, hence negligibly small. Once $W(z)$ is determined, the complex integral I_c, hence the growth rate, can be computed as the complex part of kc that would promote the growth (or decay) of the surface elevation $\eta = a \exp[\mathrm{i}k(x - ct)]$:

$$\gamma = k\mathrm{Im}(c) = (1/2)skc_0\mathrm{Im}(I_c), \tag{3.10}$$

where $\mathrm{Im}(I_c)$ denotes the imaginary part of the complex integral I_c.

Miles (1957) defines a slightly different, dimensionless growth rate β, which may be expressed in terms of the imaginary part of the complex integral I_c as

$$\beta = (c_0/U_1)^2\mathrm{Im}(I_c), \tag{3.11}$$

where U_1 is a characteristic velocity related to the so-called friction velocity u_* by the relation $U_1 = u_*/\kappa$, κ being the von Kármán constant taken as $\kappa = 0.41$. In this work, all the wave growth values are presented according to the above definition of β.

4. Computational results and related discussion

4.1. *Comparisons with Conte & Miles (1959)*

Conte & Miles (1959) gave an accurate numerical method to compute the wave growth rates for a logarithmic wind profile of the form

$$U(z) = U_1 \ln(z/z_0), \tag{4.1}$$

in which U_1 is as defined previously and the roughness length z_0 is determined empirically, the most frequently used expression being given by Charnock (1955) as $z_0 = \alpha_{ch} u_*^2/g$ where $\alpha_{ch} \sim 0.011$–0.018 is Charnock's constant. Note that for the above

	$\Omega = 3\times10^{-3}$		$\Omega = 1\times10^{-2}$		$\Omega = 2\times10^{-2}$	
c_0/U_1	Conte–Miles	Present work	Conte–Miles	Present work	Conte–Miles	Present work
1	3.536	3.533	3.237	3.233	2.747	2.744
2	3.414	3.412	3.302	3.298	2.928	2.925
3	3.433	3.431	3.208	3.205	2.779	2.775
4	3.431	3.428	2.966	2.962	2.427	2.424
5	3.301	3.297	2.547	2.544	1.909	1.907
6	2.975	2.971	1.965	1.963	1.288	1.287
7	2.441	2.438	1.290	1.289	0.677	0.677
8	1.750	1.748	0.646	0.647	0.223	0.224
9	1.016	1.015	0.193	0.193	0.026	0.026
10	0.405	0.405	0.018	0.018	0.00024	0.00024

TABLE 1. Comparisons of the dimensionless growth rate β for a range of c_0/U_1 values and for three different Ω.

logarithmic wind profile the critical height z_c, where the wind velocity equals the phase velocity c_0, is $z_c = z_0 \exp(c_0/U_1)$.

The computational results of Conte & Miles (1959) for the dimensionless growth rate β were tabulated against c_0/U_1 for three values of the parameter $\Omega = gz_0/U_1^2$, which is directly related to Charnock's constant by $\Omega = \kappa^2\alpha_{ch}$. In table 1 the wave growth rates as obtained from the numerical approach described in §2.2 together with equation (3.8) and (3.11) are compared with the results given by Conte & Miles (1959). Although both the numerical solution technique and the method of computing the growth rates are different the overall agreement is excellent, the percentage of maximum difference being only 0.12%. Insensitivity of the computations to the choice of ε is also checked repeatedly to ensure the reliability of the results.

4.2. *Growth rates for a different velocity profile*

The approach introduced in §2.2 allows the solution of the Rayleigh equation for arbitrary wind profiles. By taking advantage of this general method, the effects of different wind profiles on the wave growth rates may readily be investigated. Besides the logarithmic profile the most commonly used wind profile is the so-called 1/7-power-law profile, which is defined as

$$U_p(z) = U_{10}(z/10)^{1/7}, \tag{4.2}$$

where U_{10} is the wind velocity at 10 m height. Since the comparisons are to be made with the results of the logarithmic wind profile in terms of the dimensionless parameters (c_0/U_1) and Ω, it is necessary to introduce slight modifications to equation (4.2) in order to make it compatible with the logarithmic profile. It is first noted that the dimensionless critical height kz_c for the logarithmic wind profile is

$$kz_c = \Omega(c_0/U_1)^{-2}\exp(c_0/U_1). \tag{4.3}$$

For meaningful comparisons the dimensionless quantities (c_0/U_1) and Ω must be the same, which, in view of (4.3), requires kz_c be the same. Equation (4.2) is then modified as

$$U_p(z) = U_{1p}(z/z_0 - 1)^{1/7} \quad \text{where} \quad U_{1p} = c_0/[\exp(c_0/U_1) - 1]^{1/7}, \tag{4.4}$$

making $U_p(z_0) = 0$ and $z_c = z_0 \exp(c_0/U_1)$ as in (4.1). In this manner, the direct dependence of (4.4) on the dimensionless parameters (c_0/U_1) and Ω is ascertained.

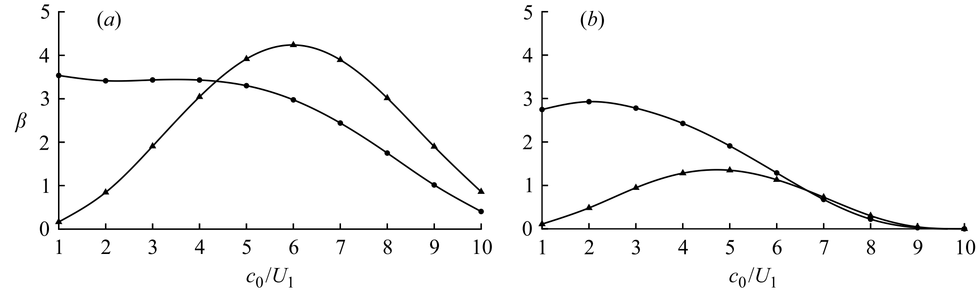

FIGURE 1. Dimensionless growth rate β as a function of c_0/U_1 for the logarithmic (●) and 1/7-power-law (▲) wind profiles: (*a*) $\Omega = 3 \times 10^{-3}$, (*b*) $\Omega = 2 \times 10^{-2}$.

Figure 1(*a, b*) depicts the dimensionless growth rates as a function of (c_0/U_1) for the logarithmic and the 1/7-power-law profiles for two different Ω values. Although the growth rates for $c_0/U_1 < 3$ would not be physically meaningful, as indicated by Conte & Miles (1959), the different characteristics of the two curves for low c_0/U_1 values are noticeable. For larger values of c_0/U_1 the characteristics of the curves become similar but the quantitative differences are obvious, especially for $\Omega = 3 \times 10^{-3}$. Since Ω is inversely proportional to the friction velocity, larger Ω values imply smaller shearing stress velocities, hence smaller growth rates. These comparisons clearly indicate the appreciable influence of the mean wind profile on the wave growth rate. For reliable growth rate estimates it becomes crucial to have measured mean wind profiles within 10 m above the sea surface. At present, the lack of alternative wind profiles based on measurements is probably the severest obstacle to progress.

5. Conclusions

Within the context of the problem of wave generation by wind, an unconventional approach is advanced for the solution of Rayleigh's instability equation for arbitrary wind profiles. Instead of attempting to develop approximate series solutions, the method, following Rayleigh (1895), introduces plausible approximations to the differential equation itself and transforms it to a Bessel's equation of the first order, which has well-known analytical solutions. As formulated, the technique is applicable to practically any wind profile selected. The accuracy of both the solution technique and the method of computing the wave growth rate is demonstrated for the special case of a logarithmic wind profile used in the computations of Conte & Miles (1959).

The general applicability of the approach makes it possible to investigate the effects of the wind profile on the wave growth rate by using wind profiles other than the almost exclusively used logarithmic one. One such example is the 1/7-power-law profile which exhibits appreciable differences in wave growth rates when compared with the logarithmic wind profile. The lack of measured data on the wind speeds within the lowest 10 m of the atmosphere, where most critical heights are located, stands as an important obstacle in forming definite conclusions on the actual effects of wind profile differences on wave growth rates.

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